

NAG Toolbox for MATLAB

s01ba

1 Purpose

s01ba returns a value of the shifted logarithmic function, $\ln(1+x)$, via the function name.

2 Syntax

```
[result, ifail] = s01ba(x)
```

3 Description

s01ba computes values of $\ln(1+x)$, retaining full relative precision even when $|x|$ is small. The function is based on the Chebyshev expansion

$$\ln \frac{1+p^2+2p\bar{x}}{1+p^2-2p\bar{x}} = 4 \sum_{k=0}^{\infty} \frac{p^{2k+1}}{2k+1} T_{2k+1}(\bar{x}).$$

Setting $\bar{x} = \frac{x(1+p^2)}{2p(x+2)}$, and choosing $p = \frac{q-1}{q+1}$, $q = \sqrt[4]{2}$ the expansion is valid in the domain $x \in \left[\frac{1}{\sqrt{2}} - 1, \sqrt{2} - 1 \right]$.

Outside this domain, $\ln(1+x)$ is computed by the standard logarithmic function.

4 References

Lyusternik L A, Chervonenkis O A and Yanpolskii A R 1965 *Handbook for Computing Elementary Functions* p. 57 Pergamon Press

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

Constraint: $x > -1.0$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, $x \leq -1.0$.

The result is returned as zero.

7 Accuracy

The returned result should be accurate almost to *machine precision*, with a limit of about 20 significant figures due to the precision of internal constants. Note however that if x lies very close to -1.0 and is not exact (for example if x is the result of some previous computation and has been rounded), then precision will be lost in the computation of $1 + x$, and hence $\ln(1 + x)$, in s01ba.

8 Further Comments

Empirical tests show that the time taken for a call of s01ba usually lies between about 1.25 and 2.5 times the time for a call to the standard logarithm function.

9 Example

```
x = 2.5;
[result, ifail] = s01ba(x)

result =
    1.2528
ifail =
     0
```